BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B70 09049

SUBJECT: Synchronized Earth Parking Orbits for Zero-Plane-Change Transfer Between Earth's Surface and Polar Lunar Orbits - Case 105-4

DATE: September 18, 1970

FROM: C. S. Rall

ABSTRACT

Low circular Earth parking orbits that are synchronized to allow passage over the launch site at least once every several days and to allow frequent zero-plane-change transfers to a specified lunar polar orbit are determined. This memorandum presents 40 orbits which have altitudes less than 300 nmi, lunar orbit opportunities at least once every six months, and passage over the launch site at least once every three days. A few of the 40 orbits are impractical because of very low altitudes or inclinations. Eighteen orbits result in Earth Orbit Shuttle (EOS) payloads greater than that to the Space Station orbit at 270 nmi and 55°.

(NASA-CR-113624) SYNCHRONIZED EARTH PARKING ORBITS FOR ZEROPLANE-CHANGE TRANSFER BETWEEN EARTH'S SURFACE AND POLAR LUNAR ORBITS (Bellcomm, Inc.) 15 p

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MEMORANDUM FOR FILE

INTRODUCTION

Earth parking orbits for lunar shuttle fueling and payload assembly will be required in future lunar missions. These orbits should be sufficiently low in altitude and inclination so that the Earth Orbit Shuttle (EOS) can bring large payloads to them but high enough in altitude for reasonable orbit lifetimes. In addition, it would be desirable for vehicles in the Earth parking orbits to pass over the launch site at least once every several days so that the EOS has frequent direct (no plane-change or phasing) rendezvous opportunities. Another desirable property is that these orbits have frequent opportunities for zero-plane-change missions to a specified lunar orbit. This second property requires synchronization of the Earth orbit regression rate with the Moon's angular motion. Such missions have translunar injection (TLI) and lunar orbit insertion (LOI) maneuvers that require no ΔV penalties for plane change.

If the Moon's orbit and the equatorial planes of the Earth and Moon were all coplanar, the Earth orbit solutions found below would be compatible with zero-plane-change return from the lunar orbit for a properly selected lunar orbit stay time. However, because of the actual Earth-Moon geometry, repeating zero-plane-change round-trip missions cannot be expected to exist.

The ability to go to a specified lunar orbit is required if it is necessary to rendezvous with a lunar orbit space station. Selection of a polar lunar orbit would allow complete landing site coverage (without plane change). The lunar mission might involve payload delivery or resupply of the station and a lunar landing craft.

Under other conditions, initially reaching any lunar orbit would be acceptable; and the primary requirement would be frequent EOS launch opportunities. One such condition would be that the lunar shuttle has the capability for large plane changes at the moon in order to reach the lunar orbit station. Another possible condition would be if the missions are direct to the lunar surface with no rendezvous with a lunar orbit space station.

Several simplifying assumptions are made here. All parking orbits are assumed circular, and the specific characteristics of the Earth-Moon transfer trajectory are ignored. Abort constraints are also ignored. The plane of the lunar orbit is assumed to be nonrotating in inertial space. With a triaxial Moon (all moments of inertia different) this assumption holds true only for a polar orbit.[1] Orbit planes of other inclinations do not remain fixed and require modification of the analysis given below.

The Earth orbit inclination must be equal to or greater than the latitude (28.5°) of the launch site (Cape Kennedy) in order to pass over the launch site at all. Solutions for lower inclination orbits, however, are included for completeness.

The disturbance acceleration on an Earth orbit vehicle due to the non-spherical figure of the Earth causes the longitude of the ascending node to regress while the inclination and altitude of the orbit remain unchanged. (The longitude of the ascending node specifies the point where the orbit passes over the equator in a northerly direction). Battin[2] gives the <u>prog</u>ression rate $\dot{\Omega}$ for a circular Earth orbit, relative to a non-rotating coordinate frame as,

$$\dot{\Omega} = -10 \left(\frac{R}{R + h}\right)^{3.5} \cos i \quad \text{deg/day} \tag{1}$$

where R is the Earth's equatorial radius, h is the orbit's altitude, and i is the orbital inclination. Contours of constant regression rate are shown in Figure 1 as plots of altitude versus inclination. Earth orbits which <u>pro</u>gress ($\dot{\Omega}>0$) also exist but are retrograde ($i>90°$). However, an EOS can deliver little or no payload into retrograde orbits (from KSC launch), and they will not be considered further.

LUNAR LAUNCH OPPORTUNITIES

For a zero-plane-change departure from Earth orbit, the transfer trajectory plane must contain the Earth orbit at departure; and to reach the Moon, it must contain the Moon at the time of arrival. Launch opportunities occur a short time (i.e., the translunar transfer time) before the Moon passes through the Earth orbit plane. This situation repeats after the Moon and the Earth orbit plane have traveled roughly 180° relative to each other. Successive opportunities, however, have transfer trajectories which alternately ascend and descend relative to the Earth's equator and therefore have different velocities relative to the Moon. In order to consider exactly repeating lunar approach geometries, 360° is taken as the angle of travel between opportunities. The relative angular rate is $\eta - \dot{\Omega}$, where η is the mean angular motion of the Moon of 13.2°/day. Hence the opportunities to reach the Moon repeat at intervals.

$$\tau = \frac{360°}{\eta - \dot{\Omega}} \tag{2}$$

A desire to enter a given polar orbit at the Moon with zero-plane-change restricts one's choice of Earth orbit. A vehicle on an Earth-Moon trajectory will approach the Moon at some hyperbolic excess speed and some angle with the Earth-Moon line. Figure 2 shows the angle as θ. This vector must lie in the lunar orbit plane if no plane change is to occur. Figure 2 illustrates the two times in a lunar month when a zero-plane-change entry into the lunar orbit is possible. Only one of these times is considered so that the Moon will be the same elevation above the Earth equatorial plane and the geometry will repeat exactly. (The lunar month used here is 27.3 days in length and is related to inertial space indicated by distant stars.)

In summary, a zero-plane-change transfer from an Earth orbit to a given lunar polar orbit has two requirements. The Earth-Moon transfer trajectory plane (also the Earth orbit plane) at TLI must contain the Moon at LOI, and the lunar polar orbit plane at LOI must be at a specified angle to the Earth-Moon line. The first requirement repeats at intervals of the time τ (given by Equation (2)). The second requirement repeats at intervals of a lunar month. In order for the two requirements to occur simultaneously at regular intervals, an integer number k of τ's must be equal to an integer number j of lunar months or

$$\frac{k}{j} = \frac{360°/\eta}{\tau} = 1 - \frac{\dot{\Omega}}{\eta} \tag{3}$$

Hence, the requirement that j and k be integers is equivalent to the requirement that $\eta/\dot{\Omega}$ be a rational number. This regular matching of the requirements is illustrated in Figure 3 with k=2 and j=3.

Choice of the quantity k/j (or $\eta/\dot{\Omega}$) specifies both τ and $\dot{\Omega}$. $\dot{\Omega}$ determines a curve of h vs i in Figure 1, specifying a family of Earth orbits. The 9 dotted curves correspond to j's (lunar months) between 1 and 6 for intervals between opportunities up to 6 months. The k's (lunar opportunity intervals) chosen are limited by the maximum value of $|\dot{\Omega}|$ of 10°/day for an orbit of zero altitude and inclination.

If the Earth's equatorial plane, the lunar orbit plane, and the lunar equatorial plane were the same, then the angles indicated in Equations (2) and (3) would be 180° instead of the 360° shown. One would not have to distinguish between travel ascending or descending relative to the common plane. In this case, zero-plane-change opportunities would occur at intervals of j/2 lunar months. With the actual Earth-Moon geometry, however, alternate opportunities require a non-zero plane change. The size of this plane change depends on many factors and is beyond the scope of this study.

EARTH TO EARTH-ORBIT LAUNCH OPPORTUNITIES

As an orbiting vehicle travels around its orbit, the Earth rotates under the orbit and the orbit plane regresses due to the Earth's oblateness. Relative to a point on the Earth's surface, the orbit plane makes one circuit in the time,

$$T = \frac{360°}{\omega_{ie} - \dot{\Omega}} \tag{4}$$

where ω_{ie} = 360.99°/day is the Earth's inertial rotation rate. (T is approximately one day, because $\dot{\Omega}$ is much smaller than ω_{ie} in magnitude.)

The angular motion in the orbit must be synchronized with T for an orbiting vehicle to pass over the launch site every few days. The mean angular motion in the orbit is given approximately by,

$$\omega = \sqrt{\frac{\mu}{(R + h)^3}} = 6136 \left(\frac{R}{R + h}\right)^{1.5} \text{deg/day} \tag{5}$$

where μ is the gravitational constant of the Earth. If passage over the launch site at least every m days is desired, then one must have,

$$m\ T = 360°\ n/\omega \tag{6}$$

where n is the number of orbital periods and both n and m are integers.

If m and n have been divided by their greatest common divisor and if passes over the launch site at only one azimuth are considered, then the time between successive launch site passes is given by mT. (The launch latitude intersects the orbit trace at two points; only one is considered, because, in general, the phasing in the orbit can be correct at only one.) Figure 4 (from Reference 3) shows curves of h vs i for families of orbits which meet the conditions. Figure 4 is restricted to orbits which have $i \leq 90°$ and $m \leq 3$ corresponding to a maximum of about 3 days between launch opportunities. (Reference 3 gives curves for $m \leq 7$.) Orbits with $h < 100$ nmi may be impractical because of their short lifetimes.

SOLUTION ORBITS

Orbits which have both frequent passage over the launch site and frequent lunar launch opportunities are the solution orbits. They occur at the intersection of the dotted curves from Figure 1 with curves from Figure 4. Table 1 presents the solution orbits that pass over the launch site at least once every three days, that supply lunar opportunities at least once in six months, and that have $i \leq 90°$ and $h \leq 300°$ nmi. The orbits are indicated by altitude (above the equator) in nautical miles and inclination in degrees. Each solution is also characterized in Table 1 by $\dot{\Omega}$ in deg/day, the time in lunar months (j) between opportunities to reach a specific lunar polar orbit, and the values of m and n (the number of days and the number of orbital periods respectively between Earth-to-orbit launch opportunities). X's in the table indicate that the corresponding solution orbits do not exist.

Several things are worth noting about specific orbits given in Table 1. Most of the solutions in the columns containing X's are impractical due to low inclinations. Several orbits in the bottom row may be impractical because of their low altitudes. For a given Earth-to-Earth orbit frequency, the highest payload solutions are in general those that have bi-monthly opportunities. The orbit at 270 nmi and 55.0° is the space station orbit.

Additional solutions can be found by relaxing the restrictions on altitude, inclination, lunar opportunity frequency, and/or Earth orbit launch frequency. Orbits with less frequent opportunities to reach a given lunar orbit have altitudes and inclinations between those of orbits with more frequent opportunities. Orbits that pass over the launch site less often also occur with $h < 300$ nmi and $i \leq 90°$. Other solutions at higher altitudes and inclinations have opportunities as frequently as those presented in the table.

If a specific lunar orbit is not desired, solutions are not constrained to being multiples of a lunar month. The only restrictions then are the Earth orbit launch restrictions indicated by the curves of Figure 4. Equations (1) and (2) give the approximate time $\tau/2$ between opportunities to reach the moon from an orbit indicated by a point in this figure.

Figures 5 and 6 superimpose solution points on shuttle payload capability contours (of h vs i) from Reference 4. Figure 5 represents a 25 Klb payload shuttle and Figure 6, a 50 Klb EOS. These payloads (25 and 50 Klb) correspond to a round trip to the space station orbit. The frequency of opportunities for solution points may be obtained by referring to Table 1. Eighteen of the solutions listed have inclinations greater than the latitude of the launch site (28.5°), have altitudes greater than 100 nmi, and allow the EOS to carry more payload than that to the space station orbit. The orbit at 101 nmi and 43° allows the 25 Klb EOS to deliver 47 Klb and the 50 Klb EOS, 72 Klb. It also passes over the launch site daily and has a lunar orbit opportunity once every other month. One more solution allows a large payload but has an altitude less than 100 nmi.

SUMMARY

Many Earth orbits are available that offer frequent zero-plane-change opportunities to travel to a particular lunar polar orbit and that also pass over the launch site

at least once every several days. Many of these allow a greater EOS payload than does the space station orbit. Practically any orbit can be used if Earth and lunar launch time intervals are sufficiently long.

C. S. Rall

1013-CSR-klm C. S. Rall

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TABLE 1. EARTH ORBIT SOLUTIONS (h IN NMI/i IN DEG)

m	n	time between lunar polar orbit opportunities in lunar months	3	5	2	5	3	4	5	6	1
		$\dot{\Omega}$ in deg/day	-8.78	-7.91	-6.59	-5.27	-4.39	-3.29	-2.64	-2.20	0
1	15		X	X	258/31.8	265/46.7	270/55.0	277/64.3	281/69.6	284/73.1	298/90.0
3	46		X	197/15.9	204/36.2	211/49.4	216/57.0	223/65.7	227/70.7	230/74.0	244/90.0
2	31		X	170/20.3	177/38.1	185/50.6	190/57.9	196/66.3	200/71.2	203/74.4	217/90.0
3	47		X	145/23.9	152/39.8	159/51.8	164/58.8	170/67.0	174/71.7	177/74.8	191/90.0
1	16		90/15.8	94/29.5	101/43.0	108/53.9	113/60.4	120/68.1	124/72.6	127/75.5	141/90.0

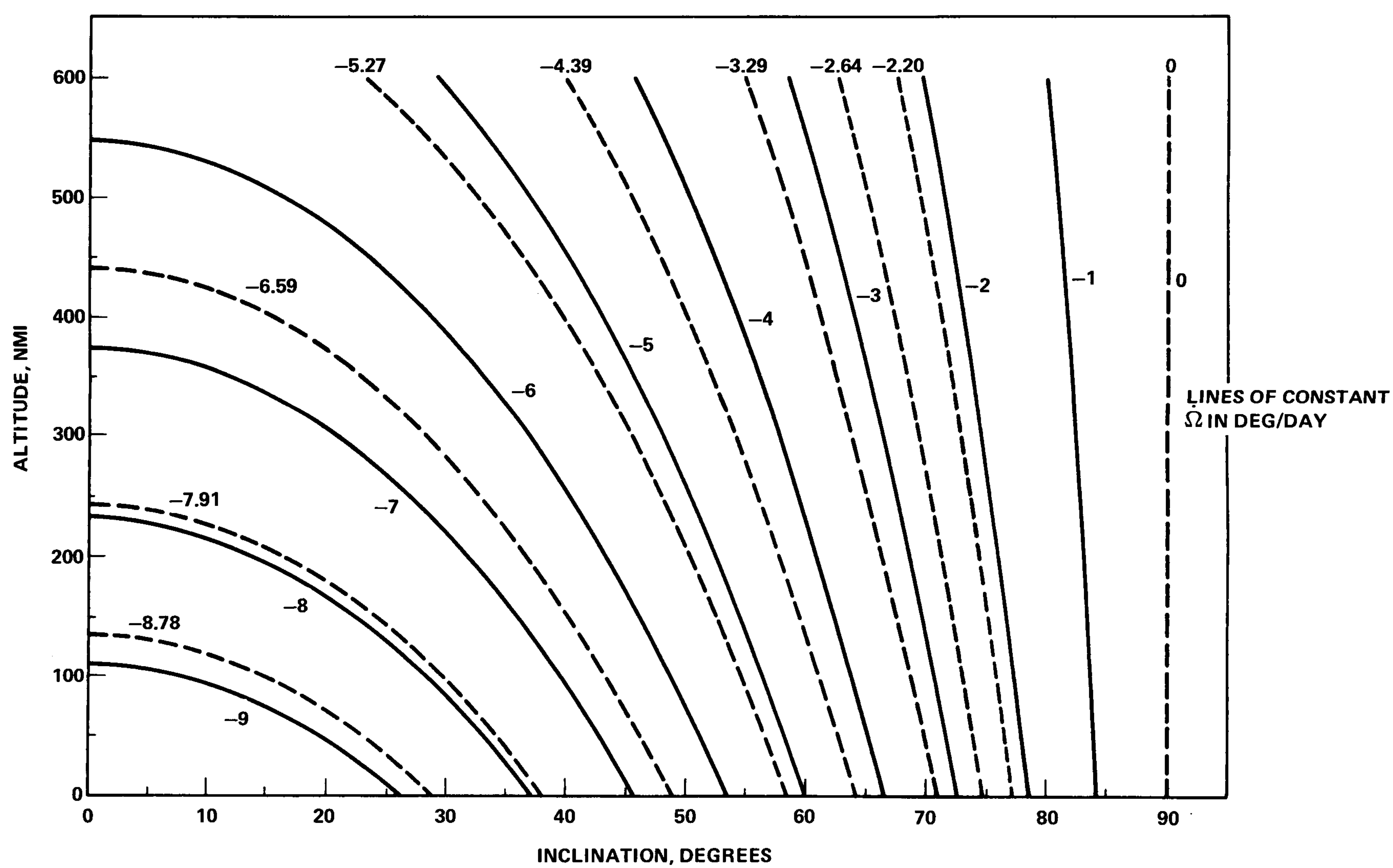

FIGURE 1 - CONTOURS OF CONSTANT ORBITAL PROGRESSION RATE

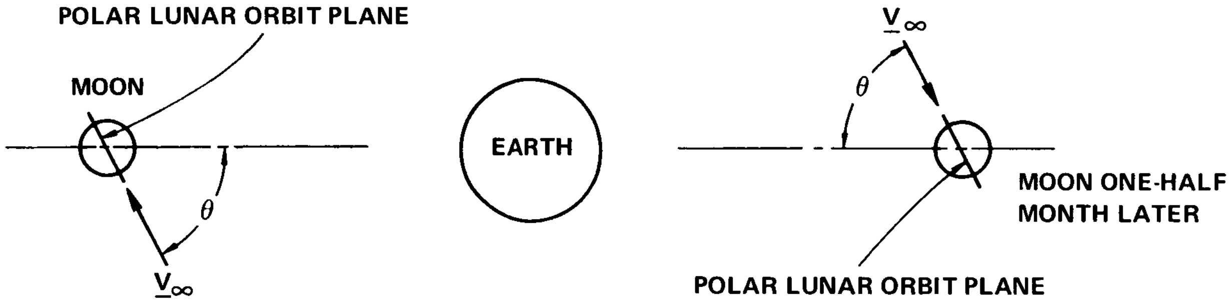

FIGURE 2 - MOON APPROACH FOR ZERO-PLANE-CHANGE

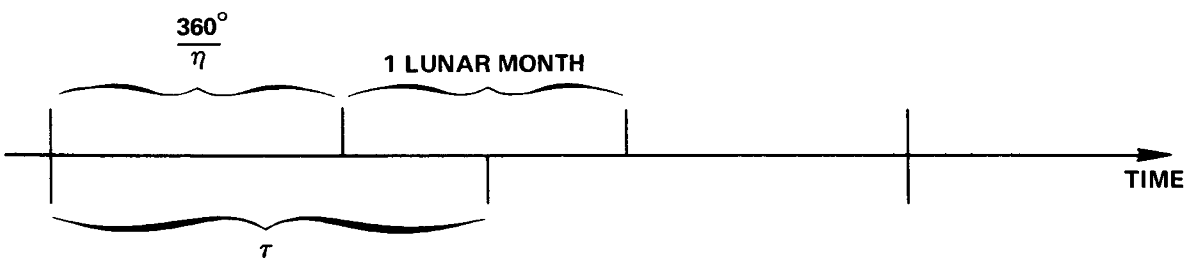

FIGURE 3 - MATCHING OF τ WITH LUNAR MONTHS

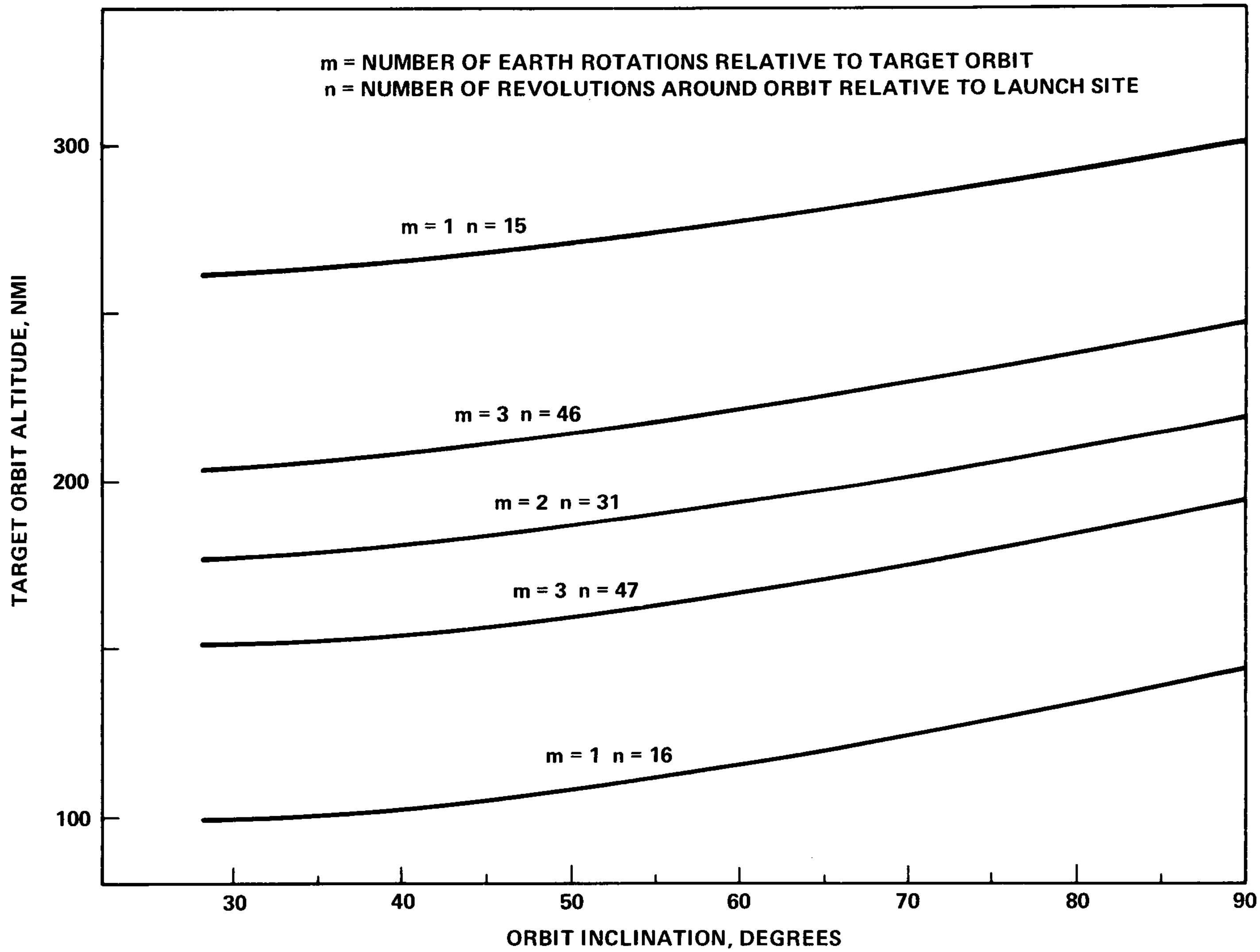

FIGURE 4 - RENDEZVOUS COMPATIBLE ORBITS — TARGET ORBIT ALTITUDE VERSUS ORBIT INCLINATION

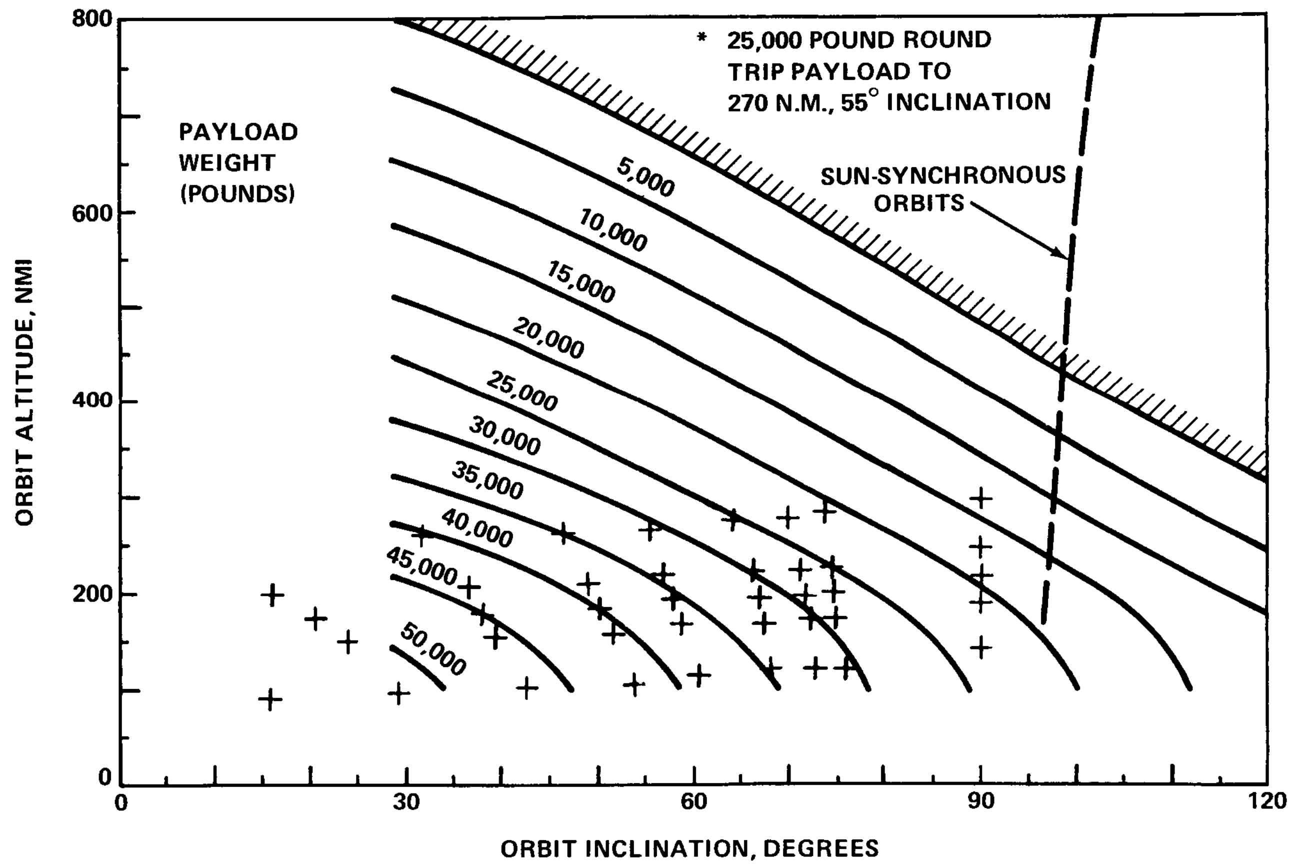

FIGURE 5 - ESTIMATED PAYLOAD PLACEMENT CAPABILITY OF A 25,000 POUND PAYLOAD*
SPACE SHUTTLE – DIRECT INJECTION, DIRECT DEORBIT.

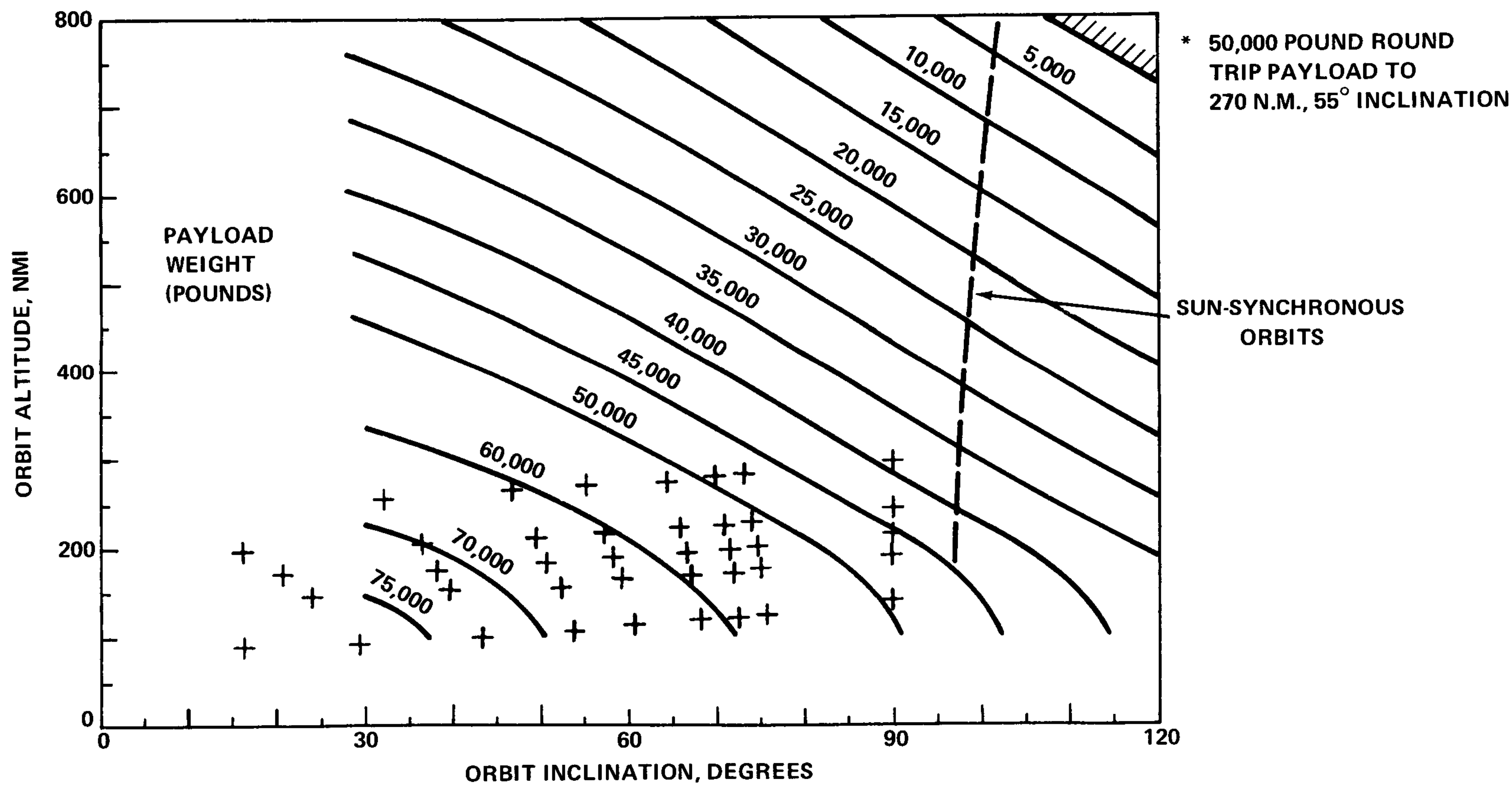

FIGURE 6 - PAYLOAD PLACEMENT CAPABILITY OF A 50,000 POUND PAYLOAD*
SPACE SHUTTLE – DIRECT INJECTION, DIRECT ORBIT

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